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- First assignment up tomorrow, due next Thursday.
- e-mail instructor button

Back to Limits

Basic Fact: $\lim_{x \to a} x = a$

for all real numbers a.

Here, $F(x) = x$

$L = a$ (L = limit)

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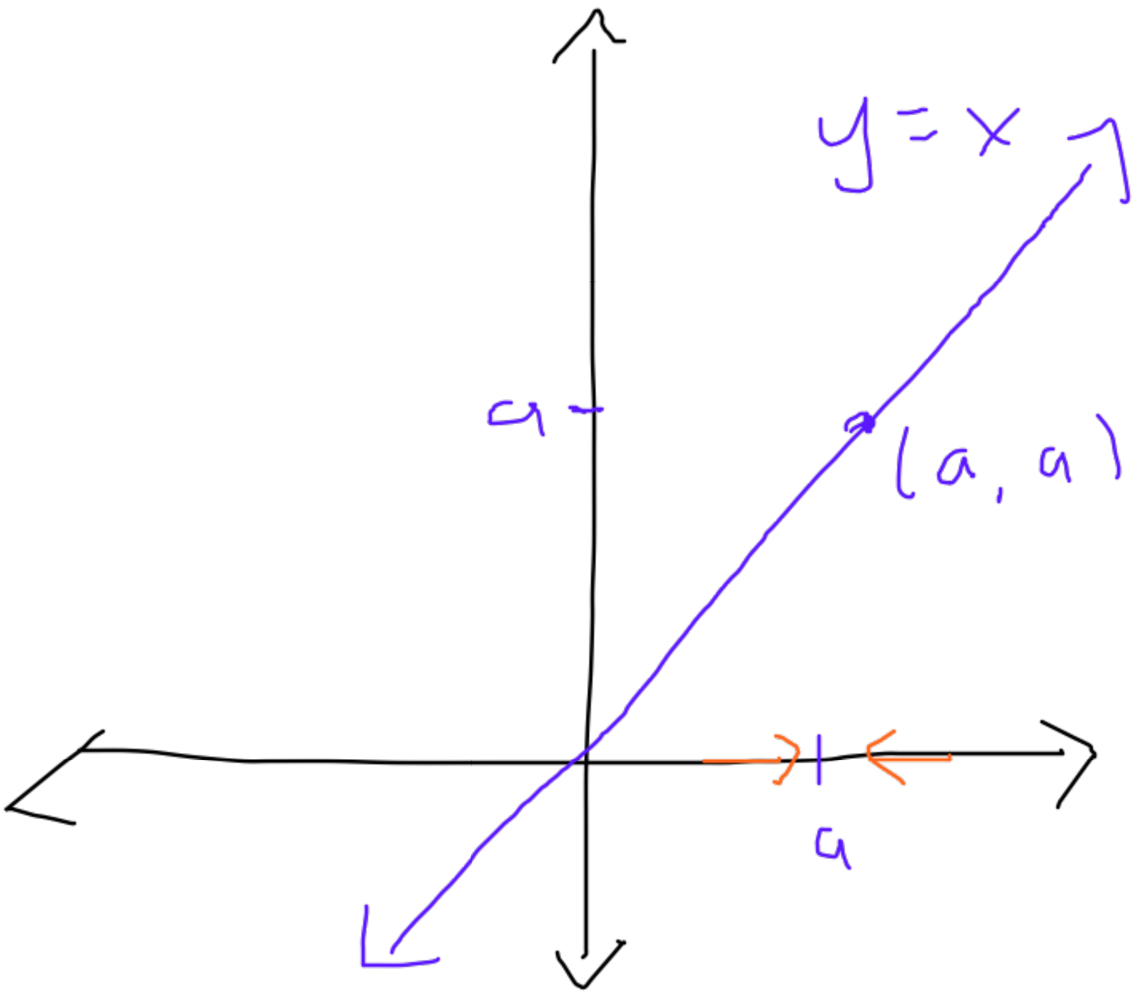
Picture



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Limit laws

Suppose $\lim_{x \to a} f(x) = L$

and $\lim_{x \to a} g(x) = M$.

a) $\lim_{x \to a} (f(x) + g(x))$

$= M + L$

b) $\lim_{x \to a} (f(x) - g(x))$

$= L - M$

c) $\lim_{x \to a} (f(x) \cdot g(x)) = L \cdot M$

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d) $$\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{L}{M}$$

$M \neq 0$

e) If c is any real number, then

$$\lim_{x \to a} c \cdot f(x) = c \cdot \lim_{x \to a} f(x)$$

Applying limit laws

$$\lim_{x \to a} x^2 = \lim_{x \to a} (x \cdot x)$$

$$= a \cdot a = a^2$$

Similarly,

$$\lim_{x \to a} x^n = a^n .$$

Using the limit laws, we get that

1) If $p(x)$ is any polynomial, then

$$\lim_{x \to a} p(x) = p(a).$$

2) If $q(x)$ is any other polynomial and $q(a) \neq 0$, then

$$\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$$

Idea: To find

$\lim_{x \to a} f(x)$, most of the time you just plug in the number to $f(x)$. Where this doesnt work: division by zero, piecewise-defined functions.

Examples

1) $\lim_{x \to 4} (22x^2 + 10x + 1)$

$= 22(4^2) + 10 \cdot 4 + 1$

$= \boxed{393}$

2)

$$\lim_{x \to 8} \frac{7x-5}{x^2+9}$$

$$= \frac{7 \cdot 8 - 5}{8^2+9}$$

$$= \boxed{\frac{51}{73}}$$

This works since the denominator isn't zero.

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with zero:

$$\lim_{x \to 0} \frac{7x-5}{x^2+9}$$

$$= \frac{-5}{9}$$

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3) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

plugging in $x = 1$ gives $\frac{0}{0}$, which is nonsense.

$\frac{0}{0}$ always means more work!

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Try factoring, remember that limits don't care about the actual point $x=a$.

$$\frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

$$= x+1$$

So $\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} (x+1)$

$$= \boxed{2}$$

4)

$$f(x) = \begin{cases} 2x-3, & x > 4 \\ 19, & x = 4 \\ x^2+x-15, & x < 4 \end{cases}$$

Find $\lim\limits_{x \to 4} f(x)$.

You have to check both

$\lim\limits_{x \to 4^-} f(x)$ and $\lim\limits_{x \to 4^+} f(x)$.

$\lim_{x \to 4^-} f(x)$ means $x < 4$.

If $x < 4$, then

$f(x) = x^2 + x - 15$, so

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (x^2 + x - 15)$$

$$= \boxed{5}$$

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$\lim_{x \to 4^+} f(x)$, this means

$x > 4$. If $x > 4$,

$f(x) = 2x - 3$. So,

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (2x - 3)$$

$$= \boxed{5}$$

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Since $\lim_{x \to 4^+} f(x)$

$= \lim_{x \to 4^-} f(x)$

$= 5$,

$$\boxed{\lim_{x \to 4} f(x) = 5}$$

__If__ $\lim_{x \to 4^+} f(x) \neq \lim_{x \to 4^-} f(x)$,

then the limit does not exist.

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5) $\lim_{x \to 6} \frac{x-6}{|x-6|}$

Plugging in $x=6$

gives $\frac{0}{0}$ = more work!

$$|x-6| = \begin{cases} x-6, & x \geq 6 \\ -(x-6), & x < 6 \end{cases}$$

Piecewise defined!

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$$\lim_{x \to 6^+} \frac{x-6}{|x-6|}$$

$$= \lim_{x \to 6^+} \frac{x-6}{x-6} = 1$$

$$\lim_{x \to 6^-} \frac{x-6}{|x-6|}$$

$$= \lim_{x \to 6^-} \frac{x-6}{-(x-6)} = -1$$

These numbers aren't the same

So limit does not exist.

Moral: Absolute values are tricky!